Quantum Structures of a Model-Universe: Questioning the Everett Interpretation of Quantum Mechanics

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Abstract Our objective is to demonstrate an inconsistency with both the original and modern Everettian Many Worlds Interpretations. We do this by examining two important corollaries of the universally valid quantum mechanics in the context of the Quantum Brownian Motion (QBM) model: "Entanglement Relativity" and the "parallel occurrence of decoherence." We conclude that the highlighted inconsistency demands that either there is a privileged spatial structure of the QBM model universe or that the Everettian Worlds are not physically real.

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1 Introduction

"Today, it is often said that in Everettian quantum theory the notion of parallel 'worlds' or 'universes' applies only to the macroscopic worlds defined (approximately) by decoherence. Formerly, it was common to assert the existence of many worlds at the microscopic level as well. Without entering into any controversy that might still remain about this, here for completeness we shall address the Claim for both 'microscopic' and 'macroscopic' cases." (A. Valentini, Chapter 16 in Ref. [1].)

As the Everettian quantum theory claims to be a valid interpretation of the universally valid quantum mechanics, there should be no known model in which it fails to perform.

Using the Quantum Brownian Motion (QBM) model [2-5] as a 'model universe', Entanglement Relativity (ER) [6-15], and the Parallel Occurrence of Decoherence (POD) [16], we will demonstrate for the first time using these methods, an inconsistency with both the original [17] and modern version [1] of Everettian quantum theory.

The QBM model affords us the opportunity to avoid getting involved with extending or arguing in the realm of "interpretation" because firstly it is a non-trivial model universe (a model of an isolated quantum mechanical system) referring to a realistic and mathematically well-defined physical situation, and secondly, the QBM model is paradigmatic for the Everett Interpretation (cf. e.g. S. Saunders, Introductory chapter, D. Wallace, Chapter 1, and J. Hartle, Chapter 2 in Ref. [1]): as it directly implements the quasi-classical dynamics of the decoherence-selected basis of wavepackets approximately localized in both position and momentum of the open system (of the Brownian particle).

[Our emphasis] [18]: *"...it is only with the rise of* decoherence theory *that philosophers and physicists were able to recognize what was clear to Everett and to his follower DeWitt from the start, namely, that the 'quantum formalism is capable of yielding its own interpretation' and that* no additional amendments to the theory are required *in order to answer how these branches are to be defined."*

Entanglement Relativity, as introduced and discussed in Section 2, demonstrates the inconsistency with Everett's original "relative state" interpretation [17] in Section 3. The Parallel Occurrence of Decoherence introduced and discussed in Section 4 discusses the modern Everett Interpretation [1] in this form of decoherence again using the QBM model and again obtaining an inconsistency with Entanglement Relativity.

The more modern Everett Interpretation utilizes decoherence as a mechanism that enables quasi-classical structures to emerge from the underly-

ing quantum theory and which bear mutually practically independent autonomous dynamics [1]. On the other hand, the parallel occurrence of decoherence for the QBM model [16] points out the simultaneous existence of at least *two* such quasi-classical, mutually irreducible (i.e., mutually non-derivable from each other by further decomposing, grouping or permutations of subsystems) structures, each bearing its own autonomous dynamics. In other words: POD points out the existence of not only one, but *two* mutually independent and irreducible Brownian particles that are subsystems of the same composite system. As long as this is a consistent quantum mechanical picture, we show that this makes for the apparent inconsistency in the very foundations of the Everett Interpretation. In Section 5 we show that the inconsistency can be removed if there is a privileged spatial structure of the model-universe (only one Brownian particle is physically *realistic*). The absence of a particular rule/prescription or a criteria for choosing the preferred structure forces us to conclude that the highlighted inconsistency is not removed. IN NEW SECTIOM, WE EMPHASIZE THAT THE MODEL WE CONSIDER IS NOT SUSCEPTIBLE TO THE 'HIGHER-LEVEL ONTOLOGY' OF SOME MODERN ELABORATIONS OF THE EVERETT INTERPRETATION. In Section 6 we present a discussion mainly devoted to the novelty of our considerations. We conclude in Section 7.

2. Entanglement relativity

Below, we will discuss Entanglement relativity (ER) as a subtle, and perhaps often overlooked aspect of the universally valid quantum mechanics. With an eye towards this subtlety, references [6-15] demonstrate how ER appears in quantum mechanics related articles, For the sake of clarity we will now highlight and analyse its important aspects.

The hydrogen atom is defined as a bipartite system "electron + proton $(e + p)$". However, in practice it is recognized as a pair "center of mass + relative coordinates $(CM + R)$". These two atom decompositions (structures) are mutually linked by the well known linear canonical transformations (LCTs) which introduce CM and R to the atom. The relevant LCTs allow for the "separation of variables" and for the exact solution to the Schrödinger equation in the $CM + R$ decomposition. The related quantum state (while neglecting the atomic spin) is in separable form $|\chi\rangle_{CM}|nlm_l\rangle_R$, where n, l, m_l are the well known numbers from quantum theory for the hydrogen atom.

On the other hand, the Coulomb interaction between e and p leads to the conclusion that the pair $e + p$ must be in an entangled state of the form $\sum_i c_i|i\rangle_e|i\rangle_p$. Bearing in mind that $e + p$ =atom= $CM + R$, the universally valid quantum mechanics implies the following equality (for an instant of

time):

$$\sum_i c_i |i\rangle_e |i\rangle_p = |\chi\rangle_{CM} |nlm_l\rangle_R. \tag{1}$$

Eq. (1) is paradigmatic for ER in that a change of the spatial degrees of freedom of a composite system typically results in a formal change of the composite system's quantum state. There is entanglement for every instantaneous quantum state of a composite quantum system and the very concept of entanglement is *relative*.

In a more abstract form eq. (1) reads as follows, where, of course, $1+2 = C = S + S'$:

$$\sum_i c_i |i\rangle_1 |i\rangle_2 = |\chi\rangle_S |\psi\rangle_{S'}, \tag{2}$$

Then, one may undertake the task of kinematically transforming the left hand side into the right hand side of eq. (2), and *vice versa*. Generally, this is a formidable task not yet very well known. However, for some models [3-5, 8, 11], we already know about the validity of eq. (2) as a corollary of the universally valid quantum mechanics.

Equation (2) applies to a system $\mathcal{C}$ of arbitrary complexity. To illustrate, one can first directly generalize ER as follows:

$$\sum_p \alpha_p |\varphi_p\rangle_1 |\Phi_p\rangle_2 = \sum_k \beta_k |\mu_k\rangle_S |\nu_k\rangle_{S'}. \tag{3}$$

Now, two remarks are in order regarding eq. (3). First, every subsystem of $\mathcal{C}$ (1; 2; S; S') may bear its own structure and related internal entanglement. Second, the above expressions equally refer to the cases when certain subsystems (e.g. the atomic spin, or the system's environment) are neglected or implicit or that are not yet known. Thereby ER eq. (2) equally addresses the hydrogen atom as well as the quantum Universe. For the hydrogen atom in a non-relativistic frame eq. (2) is precise, while for "the Universe" the expression of eq. (2), and likewise of eq. (3), assumes that further decomposing of universal subsystems is possible.

Eq. (2) reveals the presence of nontrivial non-negligible quantum entanglement in a composite quantum system relative to the pertinent degrees of freedom. We do not account for the change in entanglement due to a change of the reference frame [19]. ER otherwise is effectively ubiquitous.

As emphasized in Appendix 1, virtually every separable state for one decomposition obtains nontrivial non-negligible entangled form for virtually every alternative decomposition despite possible exceptional cases [6, 8, 20].

For the models of the negligible entanglement see Refs. [21] and the further references therein.

Entanglement Relativity therefore is a descriptive name for that there is always an entangled form of a composite system's state.

3. Inconsistency in the Everett's "relative state" interpretation

In agreement with the now-standard considerations [1] (and the references therein), we refer to a bipartite model of the universe. This formal simplification does not bring any lack of generality to the conclusions as long as unitary dynamics (Schrödinger law) is assumed for the universe and that the "irrelevant" degrees of freedom are implicit to eq. (2) and eq. (3) for every instant of time.

3.1 The inconsistency

Everett's rule on "splitting" ("branching") [17] is simple and straightforward yet generally applicable to the measurement-like interactions in a composite system. By way of eq. (2), the "splitting" ("branching") gives rise to a World (an Everett world) that is in a separable state e.g.

$$|\varphi_p\rangle_1 |\Phi_p\rangle_2, \tag{4}$$

while the linear superposition of these states (cf. the lhs of eq. (3)) evolves in time according to the Schrödinger law such that every Everett-world is described by some separable state in the form of eq. (4). The unitary dynamics of $\mathcal{C}$ gives rise to alternating entanglement formation and "splitting of worlds". While the "original" decomposition models a quantum measurement (system 2 performs a measurement on system 1), we assume that the new system S', which appears in eqs. (2)-(3), also performs a measurement on (is in a measurement-like interaction with) subsystem S. Pending the justification of this assumption in Section 4, the alternative decomposition $S + S'$ is also eligible for "splitting".

In accordance with ER, eq. (2), eq. (3) results in:

$$|\varphi_p\rangle_1 |\Phi_p\rangle_2 = \sum_k{}' \beta_{kp} |\nu_k\rangle_S |\kappa_k\rangle_{S'} \tag{5}$$

and

$$\sum_p{}' \alpha_{pk} |\varphi_p\rangle_1 |\Phi_p\rangle_2 = |\nu_k\rangle_S |\kappa_k\rangle_{S'} \tag{6}$$

where the primed sums exhibit non-equality with the similar sums in eq. (3). The very presence of non-negligible (cf. Section 2 and Appendix 1) entanglement in eqs. (5)-(6) clearly points out: the splitting for 1+2 excludes the splitting for $S + S'$, cf. eq. (5), and *vice versa*, cf. eq. (6). Thus ER states that "splitting" *cannot* occur.

3.2 A critical note

The original Everett interpretation [17] does not consider a dynamical mechanism underlying the splitting effect. To this end, one may say that Everett's original considerations are somewhat physically 'static'. On the other hand, as emphasized in the Introduction, the modern Everett Interpretation [1] introduces the occurrence of decoherence as the fast, practically irreversible dynamical mechanism underlying the splitting without necessarily resorting to the entanglement-based arguments of Section 3.1. Due to the finite "decoherence time", the emergent quasi-classical structure of the Everett-worlds bears a "coarse-grained" time axis; for the time intervals of the order of (or shorter than) the decoherence time, the Everett-worlds are not resolved. For most practical purposes, this coarse graining of the time axis may be neglected. Bearing all this in mind, one may wonder if our conclusions may survive a more elaborate dynamical analysis.

We perform a dynamical analysis in the next section, without tackling any of the open issues such as the "preferred basis problem" [22-28] or the "probability problem" [29-33], while providing the full dynamical arguments for the existence of inconsistencies in the Everett interpretation foundation, as well as justifying the assumption of Section 3.1 alternative structure splitting eligibility. Finally, as the "consistent history approach" is a convenient language rather than a well established method for the purposes of our considerations [34], we do not present our findings in such terms.

4. Parallel occurrence of decoherence and the appearance of the inconsistency

The occurrence of decoherence for the Universe is a plausible conjecture, an extrapolation of the existing (typically very simple) models of the decoherence theory. While there is progress in describing the ever more-complex systems by decoherence, the truly complex systems, such as the Universe, are as yet out of reach. Therefore there is no alternative to modelling the Universe except but to employ relatively simple models.

Nevertheless, considering the simple models does not decrease the importance of the conclusions obtained as the Everett Interpretation has pled to full universality, i.e. to model-independence. In principle, this makes

the simple models equally useful for drawing general conclusions within the Everett Interpretation (cf. e.g. [23]).

Bearing that in mind, we examine the well-known Quantum Brownian Motion (QBM) [2-5] model, paradigmatic for our purposes. First, it is a nontrivial model referring to a realistic physical situation. Second, the QBM model directly distinguishes the Gaussian states as the decoherence-selected "preferred states" of paramount importance for the Everett Interpretation (S. Saunders, Introductory chapter in Ref. [1]):

" In contrast, states well localized in phase space–wavepackets–reliably decohere, and even though elements of a superposition, evolve autonomously from each other for a wide class of Hamiltonians. With respect to states like these, Ehrenfest's theorem takes on a greatly strengthened form. But decoherence in this sense is invariably approximate; it is never an all-or-nothing thing."

4.1 Parallel decoherence for the QBM model

We briefly consider the well-known model of quantum Brownian motion (QBM) [2-5] (and the references therein). The model considers a point-like particle 1 (or the particle's center-or-mass) interacting with the harmonic-bath oscillators (system 2). The composite system $\mathcal{C} = 1+2$ is defined by the $\mathcal{C}$'s state-space $H_1 \otimes H_2$ tensor-product structure and the total Hamiltonian:

$$\hat{H} = \frac{\hat{p}_1^2}{2m_1} + V(\hat{x}_1) + \sum_i \left(\frac{\hat{p}_{2i}^2}{2m_{2i}} + \frac{m_{2i}\omega_i^2 \hat{x}_{2i}^2}{2}\right)$$
$$\pm \hat{x}_1 \sum_i \kappa_i \hat{x}_{2i} \equiv \hat{H}_1 + \hat{H}_2 + \hat{H}_{1+2}, \tag{7}$$

where the index i enumerates the environmental particles, and the sign $\pm$ is in accordance with the variations of the model contained in the literature. The physically relevant open system models are usually considered (cf. e.g. [3]): $V(\hat{x}_1) = 0$ for the free particle, or $V(\hat{x}_1) = m_1\omega^2\hat{x}_1^2/2$ for the harmonic oscillator.

The initial state $\hat{\rho}_C$ of the pair $1+2$ is separable, $\hat{\rho}_C = \hat{\rho}_1 \otimes \hat{\rho}_{2th}$, while $\hat{\rho}_{2th}$ means that the harmonic-bath environment is in thermal equilibrium. The general QBM theory states: The open system 1 is subject to decoherence induced by its environment 2, while related "pointer basis" (the robust, quasi-classical) states are the Gaussian states. The Gaussian states dynamics is very much like that which would be expected in a classical system: due to the large environment 2, decoherence effectively irreversibly destroys the linear superpositions of the Gaussian states of the system 1; the environment 2 effectively performs the approximate position-measurement for

system 1. System 1, i.e. the "Brownian particle", exhibits quasi-classical behavior very much like the "classical Brownian particle". THE DECOHERENCE EFFECT IS QUALITATIVELY UNIVERSAL: IT DOES NOT DEPEND ON THE OPEN SYSTEM'S PARAMETERS, SUCH AS THE PARTICLE'S MASS OR FREQUENCY. Therefore the composite $\mathcal{C}$ system is a proper model-universe.

Now, analogous to the Hydrogen Atom case, we consider an alternative structure $S' + E'$ of the composite system $\mathcal{C}$, $1 + 2 = \mathcal{C} = S' + E'$: S' which represents the center-of-mass system and the E' represents the "relative positions" system for the isolated composite system $\mathcal{C}$ [16]. Thus, the two structures are *mutually irreducible*. They can not be obtained from each other by further decomposing or by related subsystems grouping. Furthermore, both open systems, 1 and S', are 'elementary' (see eq. (8) below) i.e., they have no structure themselves and can not be further decomposed into more elementary subsystems. As to the new structure, its state-space tensor-product structure is $H_{S'} \otimes H_{E'}$, while the Hamiltonian eq. (7) acquires the form:

$$\begin{aligned}\hat{H} = \frac{\hat{P}_{S'}^2}{2M} + \frac{1}{2} M\Omega^2 \hat{X}_{S'}^2 + \sum_\alpha \left(\frac{\hat{p}_{E'\alpha}^2}{2\mu_\alpha} + \frac{1}{2}\mu_\alpha \nu_\alpha^2 \hat{\rho}_{E'\alpha}^2\right) \\ + \hat{V}_{E'} \pm \hat{X}_{S'} \sum_\alpha \sigma_\alpha \hat{\rho}_{E'\alpha} \equiv \hat{H}_{S'} + \hat{H}_{E'} + \hat{H}_{S'+E'}.\end{aligned} \tag{8}$$

Therefore, the two structures' unitary dynamics are mutually *independent* and *autonomous*. Further technical details can be found in Ref. [16].

Formally, the model eq. (8) is the same as model eq. (7). The differences lie in characteristic frequencies and in the presence/absence of the open systems' external fields, cf. e.g. $V(\hat{x}_1)$ in eq. (7), likewise in the internal coupling in E' (cf. the term $\hat{V}_{E'}$ in eq. (8)), while we still have to account for ER eq. (2). Actually, for the separable initial state $\hat{\rho}_C$ regarding the $1+2$ 'fundamental' structure, ER implies the presence of correlations in the $S' + E'$ alternative structure. Therefore, it does not establish that the new (E') environment is in thermal equilibrium. Generally speaking, alternative structure decoherence occurrences may be a formidable task. Fortunately, this is not the case in our QBM example.

As it is well-known [2-5], the occurrence of the Brownian effect is largely independent from the details distinguishing both models, eq. (7) and eq. (8). Particularly, the occurrence of decoherence (i.e. of the effective approximate position-measurement of the Brownian particle) is independent of the presence/absence of correlations (quantum or classical) in the initial state, of the strength of the interaction in the composite system "Brownian particle +

harmonic-oscillators-environment" or on, the so-called, form of the "spectral density" [2-5]. The formal similarity between the two models, eq. (7) and eq. (8), allows us to draw the following conclusion on the parallel occurrence of decoherence [16]:

The unitary evolution of the initial state $\hat{\rho}_C$ generated by the Hamiltonian $\hat{H}$ gives, for the different structures of $\mathcal{C}$: For as much as System 1 represents the "Brownian Particle", in the $1 + 2$ decomposition, System S' represents the "Brownian Particle" for the $S' + E'$ decomposition

The approximate position-measurement of the "Brownian particle" S' justifies our assumption on the quantum measurement, i.e., on the alternative splitting structure eligibility, Section 3.1.

4.2 The inconsistency: dynamical arguments

It is worth repeating: Both structures, eq. (7) and eq. (8), are formally equal and mutually irreducible. Both structures bear a physically clear "system-environment split" with the large environments capable of providing fast decoherence as an effectively irreversible process. As both Brownian particles, 1 and S', are elementary (one-dimensional), the number of degrees of freedom in 2 is equal to the number of the degrees of freedom in the E' environment. The initial state for both decompositions is the same, while being subject to ER. The two decoherence processes (for 1 and for S') unfold simultaneously and autonomously, i.e. independently of each other. The basis (the "preferred states") picked out by decoherence *for both open systems* is approximately e.g a "coherent-state" (a wavepacket) basis whose dynamics are quasi-classical in the sense that the behavior of those wavepackets approximates the behavior predicted for the classical Brownian particle. In apparent disagreement with Everett Interpretation, the "[for a set of the] *wavepackets, the system is isomorphic to a collection of dynamically independent simulacra of the classical Brownian particle*" (cf. Wallace, Chapter 1 in Ref. [1]), now applies not to one but to at least two mutually irreducible and dynamically independent such "collections". This disagreement amounts to an inconsistency in the very foundations of the Everett Interpretation in the following dynamical analysis.

If we return to the environments under consideration, then the model-universe state bears entanglement (cf. J. Halliwell, Chapter 3 in Ref. [1]): *"If the Everett interpretation of quantum theory is to be taken seriously, there will exist superposition states for macroscopic systems, perhaps even for the entire universe. Since such states are not observed, it is therefore necessary to explain why they go away. This question is a key part of the general question of the emergence of classical behavior from quantum theory, an issue that*

has received a considerable amount of attention (Hartle [1994] and Chapter 2 above)."

So (cf. eq. (5) in Wallace, Chapter 1 in Ref. [1]) that for the decomposition $1 + 2$ reads:

$$|\Psi(t)\rangle_C = \int dx_1 dp_1 \alpha(x_1, p_1, t)|x_1, p_1\rangle_1 \otimes |\epsilon(x_1, p_1)\rangle_2. \tag{9}$$

In the presence of decoherence, Section 4.1, $|\alpha(x_1, p_1, t)|^2$ evolves, to a good approximation, like a classical probability density on phase space for the 1 open system.

Now Section 4.1 suggests analogous equality for alternative decomposition $S' + E'$:

$$|\Psi(t)\rangle_C = \int dX_{S'} dP_{S'} \beta(X_{S'}, P_{S'}, t)|X_{S'}, P_{S'}\rangle_{S'} \otimes |\epsilon(X_{S'}, P_{S'})\rangle_{E'}. \tag{10}$$

In the presence of decoherence, Section 4.1, $|\beta(X_{S'}, P_{S'}, t)|^2$ evolves, to a good approximation, like a classical probability density on phase space for the open system S'.

According to the universally valid Quantum Mechanics, the right hand sides of the expressions (9) and (10) must be mutually equal for the fixed instant of time, t, and for the countable set of states, ER expressions (5) and (6) appear as special cases of this equality. However, this does not support the Everett Interpretation of Quantum Mechanics.

According to the Everett Interpretation [1, 17], expressions eq. (9), and eq. (10) clearly demonstrate that both structures, 1+2 and $S'+E'$, are eligible for splitting. However, according to ER, cf. e.g. eqs (5)-(6) and Appendix 1, the decoherence-induced splitting for the two structures are mutually exclusive. Thereby, while decoherence dynamically induces a separable state for the $1 + 2$ structure, simultaneously it produces entanglement for the structure $S' + E'$, and *vice versa*. So, *within the Everettian paradigm*, one directly concludes: A separable state for one structure does not last longer than the decoherence-induced splitting for the alternative structure. Bearing in mind that the decoherence is rather fast (also for the structures in Section 4.1, cf. eq. (10) in Ref. [16]) and that in the course of the occurrence of decoherence the Everett-worlds are not resolved, we *dynamically generalize* the conclusion of Section 3.1: Mutually exclusive yet simultaneous splitting processes effectively result in the impossibility of World-Splitting for both structures of the composite system.

While the basic assumption of Everett Interpretation on the separable state for every Everett-world in a virtually arbitrary instant in time is not

inconsistent either with ER (eqs. (5)-(6)), or with POD (eqs. (9)-(10)), separately, it is *not consistent* with ER *and* POD together. That, in turn, reveals the inconsistency in the very foundation of the Everett interpretation based on the assumption of the realistic Everett worlds [1, 17]. In other words, as we have just shown, the Everett Interpretation assumption on separable states for the realistic Everett-worlds (in virtually every instant of time) [1, 17] is not only non-derivable from the universally valid quantum mechanics, it is in apparent conflict with universally valid quantum mechanics.

5. In search of the 'privileged structure'

The following idea for overcoming the above highlighted inconsistency is immediate: if there is *but one* model-universe structure (e.g. $1+2$) *realistic*, then both ER, and POD, albeit formally correct, lose their physical relevance in Everett Interpretation. This means that, neither existence, nor splitting can be considered physically realistic for the alternative $(S'+E')$ structure.

While this raises some interpretational questions, in this section, we refer exclusively to the operational recipes for selecting a preferred structure of a composite quantum system.

- *Coarse-graining. Counterintuitive structures.* Some variations on "structure" have already been considered insubstantial as applied to the foundations of the Everett Interpretation. First, the "coarse-graining" operation is found not to change the general conclusions regarding the Everett Interpretation in the context of the consistent history approach [34-37]. To this end, it is apparent that the QBM structures considered in Section 4 cannot be mutually obtained via "coarse graining" as introduced and considered in [34-37]. Second, there are some more refined structural analysis that are considered interpretationally inessential, as such structures are counterintuitive [38] (and the references therein). However, this has nothing to do with our considerations. There are at least two reasons for this. On one hand, the two systems, 1, and S', play the same physical roles of the "Brownian particle" in their respective structures, i.e, there is nothing more intuitive about the 'original' Brownian particle 1 system than there is about the 'alternative' S' system. On the other hand, not all composite system structures (and respective subsystems correlations) are physically relevant. In other words, as evident in Sections 4.1-4.2, we do not consider all possible formal structures (and their respective subsystems' entanglement) to be on equal footing. On the contrary, we distinguish the structures that can, and actually do, support the occurrence of decoherence, i.e., within the Everett Interpretation, supporting the worlds-splitting. This, of course, can not be expected for arbitrary structures–the task of the "parallel occurrence of decoherence" is a

new challenging task in the foundations of the decoherence theory [16]

- *Generalized entanglement arguments.* According to the "generalized entanglement" approach [4, 39-41], we directly recognize the experimentally accessible observables in every practical situation. As a consequence, only decompositions pertaining to the accessible, preferred observables are physically important, i.e., physically relevant. All other decompositions, in *that* situation, should be considered physically irrelevant (artificial). However, while this approach is operationally sound, it refers to the open systems that should be compared to the above 1 system, for example. *But* this is *not our case.* As given by the model eq (8), the variables of the subsystems of the new structure mix the open-system's (1's) *and* the environmental (2's) degrees of freedom regarding the 'original' structure $(1+2)$. For such kind of the structures, one usually does *not* distinguish the preferred (the accessible) observables; furthermore, for the macroscopic systems, likewise the Universe itself, this is even in principle not possible [42].

We conclude these considerations with the remarks made by Zanardi's [7]:

"Without further physical assumption, no partition has an ontologically superior status with respect to any other."

as well as by Halliwell (Chapter 3 in Ref. [1]) about the isolated quantum systems:

"*However, for many macroscopic systems, and in particular for the universe as a whole, there may be no natural split into distinguished subsystems and the rest, and another way of identifying the naturally decoherent variables is required.*"

As we have not recognized a physically sound approach, or criteria for choosing the 'privileged' isolated composite quantum system spatial structure (notably for the quantum Universe), we are forced to conclude that the basic Everett Interpretation inconsistency has not yet been resolved.

NEW SECTION

Our conclusion involves subtle considerations deserving strict observation.

Our considerations on POD, Section 4.1, directly involves the 'preferred basis problem' by pointing out the simultaneous existence of the two mutually irreducible preferred basis states–one for the 'original' (the 1 system) and the other for the 'alternative' Brownian particle (the CM system). As the pointer basis is not the focus of our paper, we leave this issue alone.

Our objective is the composite system, that is, the Everett worlds (branches) that are subject to the Schrodinger law and particularly the process of splitting (branching). We apply the standard definition of the branching for both

structures, $1+2$ and $CM+R$, cf. eqs. (9)-(10). While a change in the definition of the branching process of the Everett worlds may be ALLOWED, our conclusion on inconsistency (Section 4.2) remains intact as long as the following assumptions are fulfilled: (i) as a physical system, every Everett world (a branch) can be described by a definite instantaneous quantum state, and (ii) an Everett world quantum state is a pure, tensor-product state. As Appendix A exhibits, our conclusion remains valid also for the relaxed condition (ii), i.e. for the weakly-entangled states.

In certain elaborations of the Everett interpretation such as with Wallace's (Chapter 1 in Ref. [1]), the irrelevance of the instantaneous quantum states of the branches is implied. As a consequence, our ER-argument may seem pointless and that our conclusion is not warranted. This kind of interpretation-based reasoning calls for some higher-order ontology and 'emergent properties' (the "patterns") of the composite (complex systems). Interestingly enough, this modern thinking in terms of the Everett interpretation does not change our conclusions as it is not applicable to our QBM model, as follows.

The QBM model refers to a one-dimensional Brownian particle, while the structures considered are mutually irreducible and incomparable thus not allowing a definition of an 'emergent Brownian particle'. Let us carefully emphasize this.

The Brownian particle is a one-dimensional system. It directly implies two things. First, there is no room for complexity (for the 'patterns') of a single one-dimensional system. Second, employing the fuzzy boundary (i.e. the 'approximate decoherence') between the open system and its environment a la Ladyman (Ref. [1], p. 156) "*... and it is quite reasonable to think that macroscopic objects are not composed of exact numbers of microparticles"* is obviously also inapplicable to the Brownian particle model. Finally, it also appears impossible to define an 'emergent Brownian particle' that should somehow incorporate both Brownian particles, 1 and CM, of Section 4.1. The reason is that the two Brownian particles, 1 and CM, are mutually *irreducible* and in a sense *mutually exclusive*. Formally: the scalar product of the respective Gaussian (preferred) states for the 1 system and the CM system is not defined. That gives rise to the 'distinguishability' of the two Brownian particles is simply meaningless as well as to the mutually exclusive information contents of the two Brownian particles, 1 and CM. As to the later: a probability density for one particle can not be integrated over in order to provide the probability density for the other Brownian particle [16]. Worse, a quantum measurement performed on one particle leads to, according to ER in Section 2, the loss of information regarding the other Brownian particle. Therefore, as apparent from Section 4.1, there does not in principle

exist a quantum measurement providing the approximate simultaneous measurements for both Brownian particles–a measurement that reveals either the 1 or CM Brownian particle. In other words: the 'emergent Brownian particle' that should simultaneously incorporate both 'microscopic' Brownian particles, 1 and CM, is simply *un-observable*. Unless we adopt a nonscientific practice of non-observability of the 'emergent' systems, we are forced to conclude that 'emergent Brownian particle' for Section 4.1 is physically non-realistic.

Now we can re-emphasize the scope of our considerations that precisely define our conclusion as of Section 4.2: we are not interested in any general considerations regarding the Everett interpretation; rather, our task is to investigate the Everett interpretation [1, 17] in the context of the particular QBM model. It is worth emphasizing: we do not prove non-existence of the 'approximate structures' and of the related decoherence processes for the QBM model in principle. What we obtain refers to the structures introduced in Section 4.1. As we find the higher-ontology 'patterns' (e.g. the 'approximate decoherence') inapplicable to the QBM-setup structures of Section 4.1, we resort to the standard definition of the worlds-splitting (expressed in the terms of the instantaneous states of the 'Branches') and the conclusion as emphasized in Section 4.2.

6. Discussion

NOW, WE CAN GIVE AN ALTERNATIVE PROOF OF THE MAIN RESULT OF SECTION 4.2 AS FOLLOWS.

BEING MUTUALLY INDEPENDENT AND IRREDUCIBLE, cf. Section NEW SECTION, THE TWO DECOHERENCE PROCESSES FOR 1+2 AND $CM+R$ GIVE RISE TO *MUTUALLY IRREDUCIBLE (MUTUALLY NON-APPROXIMATE) SPLITTING PROCESSES* THAT–AS DISTINCT FROM THE "APPROXIMATE", EMERGENT SPLITTING DISCUSSED IN THE PRECEEDING SECTION–SHOULD BE SEPARATELY CONSIDERED.

THE UNIVERSAL QUANTUM STATE $|\Psi(t)\rangle$ PROVIDES "HISTORY" FOR BOTH STRUCTURES (AND FOR EVERY OTHER POSSIBLE STRUCTURE) OF THE COMPOSITE SYSTEM. WHEN APPLIED TO THE PARALLEL OCCURRENCE OF DECOHERENCE, SECTION 4.1, THE EVERETT INTERPRETATION RULE FOR THE WORLDS SPLITTING (BRANCHING), GIVES RISE TO THE FOLLOWING CONCLUSION: Except for the short, practically negligible time intervals, each of the order of "decoherence time", an Everett-world quasi-classically evolves in time (approximately) as mapping between separable states of form $|x_1, p_1\rangle_1 \otimes$

$|\epsilon(x_1, p_1)\rangle_2$ for $1 + 2$ structure, *and* in analogous form for $S' + E'$ structure for separable states of the form $|X_{S'}, P_{S'}\rangle_{S'} \otimes |\epsilon(X_{S'}, \hat{P}_{S'})\rangle_{E'}$. tHE SAME CONCLUSION APPEARS IN THE "CONSISTENT HISTORIES" TERMS. aCTUALLY, THE PRESENCE OF DECOHERENCE PROVIDES [Paz and ZUREK 1993 from Schloss] consistency of whole-universe histories IMPLICIT TO OUR EXPRESSIONS EQ. (9) AND EQ. (10). sO, BY PROJECTING THESE TOTAL-SYSTEM HISTORY AT TIME t, ONE OBTAINS THE STATES AND DYNAMICS (STATE-MAPPING) DESCRIBED ABOVE.

NOW, THE POINT STRONGLY TO BE EMPHASIZED IS AS FOLLOWS: EVERY EVERETT WORLD (A BRANCH) CAN BE DESCRIBED BY THE DIFFERENT, MUTUALLY DECOHERING AND SPLITTING BRANCHING STRUCTURES. This 'parallel splitting' in its APPROXIMATE form READS:

$$|x_1, p_1\rangle_1 \otimes |\epsilon(x_1, p_1)\rangle_2 \approx |X_{S'}, P_{S'}\rangle_{S'} \otimes |\epsilon(X_{S'}, \hat{P}_{S'})\rangle_{E'} \tag{11}$$

for virtually every instant in time.

However, eq. (11) is in conflict with ER. Even if in some instant(s) in time and for some special states [3-5, 8, 11] eq. (11) can be valid, it is unlikely to be satisfied in the very next instant of time as (cf. Appendix 1) the unitary evolution tends to introduce entanglement for at least one structure, $1+2$ or $S' + E'$. This was unknown to Everett [17]: Entanglement is ubiquitous for a composite system and cannot be suspended [3-12]. Consequently, dynamically, as emphasized in Section 4.2 and justified in Section 5, the 'parallel splitting' is not allowed, according to quantum mechanics.

Parallel occurrence of decoherence described in Section 4.1 is actually applicable to a number of similar models of quantum decoherence/measurement theory. Therefore it applies to a *class* of quantum-mechanically relevant models [16].

What we report on here is that the Everett Interpretation is inapplicable to at least one class of the physically relevant, realistic and *Everett-Interpretation-related* quantum mechanical models. To the extent that this result can be relied on, according to Ladyman, cf. Ref. [1], p. 154, we conclude that the Everettian Quantum Mechanics is *not "coherent"*. To the best of our knowledge, Everett Interpretation has not yet been found to depart from the universally valid quantum mechanics for any quantum mechanically relevant model.

The possible route to saving such Everett Interpretation (Section 5) demonstrates how serious is the challenge posed by our results to the foundations of the Everett Interpretation: Unless there is a privileged, spatial structure

of the Universe, the Everett interpretation appears either not to be correct or the Everett-worlds (the Everett "branches") are not physically real.

7. Conclusion

We demonstrate that the Everett Interpretation is not consistent with the universally valid quantum mechanics, as long as the Everett-worlds are considered physically realistic. This inconsistency follows from the recent results of Entanglement Relativity and the Parallel occurrence of decoherence (provided for the Quantum Brownian Motion-like models) as corollaries of the universally valid quantum mechanics. In simplified terms: the Everett worlds splitting (branching) is not allowed for the realistic Everett worlds. Thus, we conclude: Unless there is a privileged, spatial structure (decomposition into subsystems) of the model-universe, Everett Interpretation appears either to be not correct or the Everett-worlds (the Everett "branches") are not physically real. The interpretational consequences as well as some ramifications of our findings are yet to be explored.

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Appendix 1

As a criterion for separability, we employ "quantum covariance function" [11], C_f, defined as:

$$C_f = \langle \hat{A}_{S'} \hat{B}_{E'} \rangle - \langle \hat{A}_{S'} \rangle \langle \hat{B}_{E'} \rangle \tag{12}$$

that refers to a separable state $|\Psi\rangle = |\phi\rangle_1 |\chi\rangle_2$ and to a pair of the observables, $\hat{A}_{S'}$ and $\hat{B}_{E'}$, for the "new" subsystems, S' and E', respectively. If the state $|\Psi\rangle$ is also separable for the new decomposition $(S' + E')$, then ER, eq. (2), does not apply, and $C_f = 0$. Nevertheless, $C_f = 0$ is necessary, not yet a condition sufficient for state-separability.

Let us consider a pair of observables, $\hat{\xi}_{S'} = \sum_i \alpha_i \hat{x}_i$ and $\hat{\eta}_{E'} = \sum_j \beta_j \hat{x}_j$. For simplicity, let us assume that $\hat{x}_1$ is the only degree of freedom of the 1 system, and that the rest refers to the 2 system. Then, agreeing that all sums refer to the 2 system degrees of freedom, one obtains:

$$C_f = \alpha_1 \beta_1 (\Delta \hat{x}_1)^2 + \sum_i \alpha_i \beta_i (\Delta \hat{x}_i)^2 + \sum_{i \neq j} \alpha_i \beta_j (\langle \hat{x}_i \hat{x}_j \rangle - \langle \hat{x}_i \rangle \langle \hat{x}_j \rangle), \tag{13}$$

where $\Delta\hat{x}$ stands for the "standard deviations", and the averaging ($\langle * \rangle$) is over the state $|\Psi\rangle$. In order to ER eq. (2) *not* to be valid, it is necessary that $C_f = 0$ applies to all the pairs of the observables of S' and E'.

Generally, the $C_f = 0$ condition is quite non-transparent. Thus, let us consider Section 4.1 model, while–in search of the $C_f = 0$ possibility–we assume both: That the 1 system is a harmonic oscillator in its ground state, and that $|\chi\rangle_2$ is the ground-states tensor-product for every harmonic oscillator in the 2 environment. Then $\hat{\xi}_{S'} = \sum_i m_i \hat{x}_i / M$, and, e.g., $\hat{\eta}_{E'i} = \hat{x}_1 - \hat{x}_i, \forall i$, that simplifies eq. (13):

$$C_f^{(i)} = \alpha_1 (\Delta\hat{x}_1)^2 - \alpha_i (\Delta\hat{x}_i)^2. \tag{14}$$

As we deal with harmonic oscillators for the 1+2 decomposition, $(\Delta\hat{x})^2 = \hbar/2m\omega$ for each oscillator, one discovers that the desired condition $C_f^{(i)} = 0$ implies $\omega_1 = \omega_i$, where ω_1 is the 1 open system frequency, and ω_i is the ith oscillator frequency in the environment. In order for the ER not to be valid, eq. (14) forces the nonrealistic constraint $\omega_1 = \omega_i, \forall i$. So, one can say that independent of a composite system model, the number of states bearing separable form for both structures is by far negligible to the number of states fulfilling ER eq. (2). For this reason, we simply ignore possible ER exceptions in the greater portion of this paper.

The cases of ($C_f \approx 0$) approximate separability, obviously, are also accounted for above. Thus, the variables-transformations typically provide a non-trivial (non-negligible) change in quantum state separability: Typically, approximate equalities of the form $|\psi\rangle_1 |\chi\rangle_2 \approx |\Psi\rangle_{S'} |\Phi\rangle_{E'}$ are not satisfied.

From the perspective of a composite system unitary evolution, ER provides the following interesting observation: Keeping in mind, on one hand, the one-to-one correspondence between the instants of time, and the quantum state sets for the composite system, and the negligible number of the states not fulfilling ER compared to the number of states described by ER, on the other, we conclude that in the composite system unitary dynamics, for the most part, at least one structure will bear entanglement. e.g. Even if the initial state of the composite system is separable for both structures, the state dynamic change will, according to ER, spoil the separability: the dynamics spontaneously provides entanglement for at least one decomposition, already in the next instant in time.

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